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Net-zero climate policies foresee deployment of atmospheric carbon dioxide removal wit geological, terrestrial, or marine carbon storage. While terrestrial and geological storage would be governed under the framework of national property rights, marine storage implies that carbon is transferred from one global common, the atmosphere, to another global common, the ocean, in particular if storage exceeds beyond coastal applications. This paper investigates the option of carbon dioxide removal (CDR) and storage in different (marine) reservoir types in an analytic climate-economy model, and derives implications for optimal mitigation efforts and CDR deployment. We show that the introduction of CDR lowers net energy input and net emissions over the entire time path. Furthermore, CDR affects the Social Cost of Carbon (SCC) via changes in total economic output but leaves the analytic structure of the SCC unchanged. In the first years after CDR becomes available the SCC is lower and in later years it is higher compared to a standard climate-economy model. Carbon dioxide emissions are first higher and then lower relative to a world without CDR. The paper provides the basis for the analysis of decentralized and potentially non-cooperative CDR policies.

Keywords: carbon dioxide removal, climate change, integrated assessment, social cost of carbon, optimal carbon tax

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Carbon dioxide removal in a global analytic climate economy

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Abstract: Net-zero climate policies foresee deployment of atmospheric carbon dioxide removal wit geological, terrestrial, or marine carbon storage. While terrestrial and geological storage would be governed under the framework of national property rights, marine storage implies that carbon is transferred from one global common, the atmosphere, to another global common, the ocean, in particular if storage exceeds beyond coastal applications. This paper investigates the option of carbon dioxide removal (CDR) and storage in different (marine) reservoir types in an analytic climate-economy model, and derives implications for optimal mitigation efforts and CDR deployment. We show that the introduction of CDR lowers net energy input and net emissions over the entire time path. Furthermore, CDR affects the Social Cost of Carbon (SCC) via changes in total economic output but leaves the analytic structure of the SCC unchanged. In the first years after CDR becomes available the SCC is lower and in later years it is higher compared to a standard climate-economy model. Carbon dioxide emissions are first higher and then lower relative to a world without CDR. The paper provides the basis for the analysis of decentralized and potentially non-cooperative CDR policies.

Keywords: carbon dioxide removal, climate change, integrated assessment, social cost of carbon, optimal carbon tax

1 Introduction

In line with the Paris Agreement to limit global warming to well below 2°C many countries have declared their intention to transition towards a net-zero emissions economy by the second half of this century (Tanaka and O'Neill, 2018). To accomplish this goal approaches that remove carbon dioxide from the atmosphere with subsequent terrestrial, geological or marine carbon storage have been proposed (carbon dioxide removal, CDR). Furthermore, also capturing carbon at emissions point sources like industrial installations is discussed, requiring as well carbon storage options (Anderson and Newell, 2004). The process of capturing, transporting and storing carbon consumes additional energy and thus potentially leads to new emissions (IPCC, 2005). The availability of carbon dioxide removal and storage as an 'end of pipe' mitigation technology may be perceived as a substitute for conventional emission mitigation, which might lead to rebound effects (e.g. Geden et al., 2019). Furthermore, while terrestrial and geological storage would be governed under the framework of national property rights, CDR with marine storage implies that carbon is transferred from one global common, the atmosphere, to another global common, the ocean, in particular if storage exceeds beyond coastal applications.

We focus on marine carbon storage and explore the various trade-offs in a global analytic integrated assessment model where CDR technologies can be used to reduce atmospheric carbon concentrations at a cost measured in energy units. We derive the optimal level of CDR deployment and analyze how emissions, energy input, and the SCC (optimal carbon tax) are affected by the introduction of CDR. Although the model focuses on marine CDR, it is general enough to also consider the potential of Carbon Capture and Storage (CCS) technologies and CDR in different non-atmospheric boxes in general.

Atmospheric carbon dioxide only represents a small fraction of the total carbon stock in the Earth System, namely 829 gigatons (Gt) out of a total of more than 45,696 gigatons (IPCC, 2013). The rest of it is bound in other reservoirs such as the oceans that have served as an important carbon sink over the past 200 years (Sabine et al., 2004). Due to the large storage capacity, the ocean has been suggested to serve as carbon storage achieved either by direct, intentional injection of carbon dioxide via ships or pipelines (Rickels and Lontzek, 2012) or indirect increased carbon marine carbon uptake achieved by coastal blue carbon approaches, increasing marine biological productivity via fertilization achieved for example by artificial up-welling, or by increasing the chemical buffer capacity of the ocean by adding alkaline materials (ocean alkalinity enhancement).

Whether a geological reservoir, such as an exploited oil field, is well suited for CDR is mainly determined by the rate at which carbon leaks back to the atmosphere (van der Zwaan and Gerlagh, 2009). A similar problem arises, if carbon is stored in the ocean. Due to feedback and saturation effects in the carbon cycle some of the carbon that is injected into the oceans will eventually still end up in the atmosphere. Rickels et al. (2018) investigate how well these effects are captured in currently used Integrated Assessment Models (IAMs). Rickels and Lontzek (2012) explore the economic implications of the ocean's imperfect storage property. They show that optimally each ton of carbon sequestered to the ocean is taxed at a rate lower than the optimal carbon tax for atmospheric carbon emission. In this paper, We derive the SCC, which quantifies the optimal tax on carbon emissions, for different reservoir types and analyze how the optimal carbon tax is affected by the introduction of CDR technologies by comparing the results of model specifications with and without the availability of CDR.

The paper is based on the recently emerging literature on analytic IAMs which have the feature that the SCC can be written as a constant fraction of total economic output (e.g. Traeger, 2018; Gerlagh and Lsiki, 2018; Golosov et al., 2014). This result arises from specific assumptions on utility and climate change damages which ensure that the climate-economy model is linear in the model's state variables, in particular human-made capital and the stocks of carbon in the different reservoirs (Karp, 2017; Traeger, 2018). We show that due to the linear-in-states property of analytic IAMs the deployment of CDR technologies has no effect on the analytic structure of the SCC. However, CDR alters the time path of total economic output and therefore influences the level of the SCC. The quantitative analysis shows that this effect is minor and only increases the SCC by 3 USD/tCO_2 by the year 2100, where the absolute level of the SCC is around 800 USD/tCO_2.

The paper is structured as follows. The next section introduces the option of CDR in an analytic climate-economy model. Section 3 presents the theoretical results on optimal emissions, CDR deployment, and the SCC, and compares them to the outcome of a standard climate-economy model without the option of CDR. The last section provides a numerical simulation for calibrated versions of both model types.

2 Analytic climate-economy model

This section introduces the option of CDR and the storage of carbon in different reservoirs types in an analytic integrated assessment model of climate change. The model is based on Traeger (2018) and Golosov et al. (2014). We consider a global economy where gross output Y_t is a function of technology $A_{0,t}$, capital K_t, labor $N_{0,t}$, and net energy input I_t,

$$Y_t = A_{0,t}\, K_t^{\kappa}\, N_{0,t}^{1-\kappa-v}\, I_t^{v} \qquad \text{with } K_0 > 0 \text{ given.} \tag{1}$$

The subscript zero denotes that technology and labor are prescribed by time dependent exogenous processes. Subscript t denotes the point in time. We distinguish between gross energy E_t and net energy I_t, whereby only the latter enters the gross output function.

Energy E_t is derived from an exhaustible resource R_t which comprises all fossil fuels (coal, oil, and natural gas), and is measured in terms of its carbon content (in GtC). Absent a CCS technology, E_t can also be interpreted as carbon emissions. The resource stock R_t develops over time according to

$$R_{t+1} = R_t - E_t, \qquad \text{with } R_0 > 0 \text{ given.} \tag{2}$$

We follow Traeger (2018) and allow for a finite number of carbon boxes with carbon contents, M_1, $...M_r$, where $r \in \mathbb{N}$. The first carbon content, M_1, represents the atmospheric stock of carbon. The other boxes reflect the carbon stocks of the ocean and the biosphere, and potential geological storage capacities. Different to Lafforgue et al. (2008), We assume that there exist no capacity constraints for the geological reservoir. The dynamics of the carbon stocks are given by

$$\begin{pmatrix} M_{1,t+1} \\ M_{2,t+1} \\ \vdots \\ M_{r,t+1} \end{pmatrix} = \begin{pmatrix} \phi_{11} & \dots & \phi_{r1} \\ \phi_{12} & \dots & \phi_{r2} \\ \vdots & \ddots & \vdots \\ \phi_{1r} & \dots & \phi_{rr} \end{pmatrix} \begin{pmatrix} M_{1,t} \\ M_{2,t} \\ \vdots \\ M_{r,t} \end{pmatrix} + \begin{pmatrix} E_t^{net} + E_t^{\text{exo}} \\ G_{2,t} \\ \vdots \\ G_{r,t} \end{pmatrix}, \tag{3}$$

or in matrix notation

$$\boldsymbol{M}_{t+1} = \boldsymbol{\Phi} \boldsymbol{M}_t + \boldsymbol{E_t}.$$

The transition matrix $\boldsymbol{\Phi}$ shows the rates of carbon flows between the reservoir types.

CDR technologies allow to remove carbon from the atmosphere M_1, and to store it in another reservoir type M_i with i $\in \{2, ..., r\}$. The amount of additional carbon that is stored in each reservoir is measured in GtC and denoted by $G_{i,t}$. Net emissions are given by the difference between the carbon emitted during the production process and the carbon that is removed from the atmosphere,

$$E_t^{net} = E_t - G_{1,t}. \tag{4}$$

In each time period, the sum of carbon injected in all reservoir types must be equal to the amount of carbon that is subtracted from the atmospheric reservoir, thus $G_{1,t} = \sum_{i=2}^{r} G_{i,t}$. If $E_t^{net} > 0$, CDR can also be interpreted as carbon capture and storage. Net negative emissions are only present if $E_t^{net} < 0$.

The total amount of carbon that enters or leaves the atmosphere is given by the sum of net emissions E_t^{net}, and emissions from exogenous processes including land use change and forestry, which are collected in E_t^{exo} and also measured in GtC.

CDR consumes energy. It is therefore convenient to measure its operational costs $f_i(G_{i,t})$ in energy units. Thus, net energy input is the result of fossil energy net the energy used for CDR,

$$I_t = E_t - \sum_{i=2}^{r} f_i(G_{i,t}). \tag{5}$$

If a storage reservoir is not used, the corresponding costs of CDR are zero, $f_i(0) = 0$. Marginal costs are assumed to be positive and increasing for all storage units, $f_i'(G_{i,t}) > 0$ and $f_i''(G_{i,t}) > 0$. A reduction in carbon emissions into the atmosphere can either be achieved by reducing energy input E_t directly (mitigation) or by using CDR ($G_{i,t}$). Since the cost for mitigation and CDR deployment can both be measured in energy units (in GtC), reservoir i will only be used if its cost (and marginal cost) is lower than the cost (and marginal cost) of mitigation, thus $f_i(G_{i,t}) \leq G_{i,t}$, and $f_i'(G_{i,t}) \leq 1$.

Note that without the option of CDR, net energy input, emissions, and net emissions are equivalent, $I_t = E_t = E_t^{net}$.

We follow Golosov et al. (2014) and assume a direct mapping of climate change damages from the atmospheric carbon stock $M_{1,t}$. The damage function, that shows climate damage as a fraction of gross output, is given by

$$D_t(M_{1,t}) = 1 - \exp\left[-\xi_0 \left(M_{1,t} - M_1^{pre}\right)\right], \tag{6}$$

where M_1^{pre} denotes the pre-industrial atmospheric carbon concentration. The climate change damage parameter $\xi_0 > 0$ scales the marginal climate damage of atmospheric carbon, and can be reasonably calibrated to the climate damages in the DICE model (see e.g. Golosov et al., 2014).

Output net climate change damages is therefore given by $Y_t^{net} = Y_t \; [1 - D_t\,(M_{1,t})]$. The model does not include any impacts from increasing carbon concentrations in the ocean (e.g. from ocean acidification).

Following Golosov et al. (2014) we assume full depreciation of capital over the course of 10 years, the model's time step. Thus, the economy's capital stock in the next period is given as the difference between net output Y_t^{net}, and consumption C_t,

$$\begin{aligned} K_{t+1} &= Y_t \; [1 - D_t\,(M_{1,t})] - C_t \\ &= Y_t \; \exp\left[-\xi_0 \left(M_{1,t} - M_1^{pre}\right)\right] - C_t. \end{aligned} \tag{7}$$

The consumption rate is defined as $x_t = \frac{C_t}{Y_t^{net}}$, such that $1 - x_t$ is the savings rate.

We solve the model for a social planner who maximizes the infinite stream of consumption flows by choosing the consumption rate, emissions, and CDR deployment,

$$\max_{x_t, E_t, G_{i,t}} \sum_{t=0}^{\infty} \beta^t \, \log(C_t), \tag{8}$$

subject to the constraints imposed by the economy and the climate system, equations (1) to (7). The parameter β denotes the utility discount factor.

3 Theoretical results

This section presents the results of the climate-economy model, and compares them to the outcome of an alternative model specification without the option of CDR.

3.1 Carbon dioxide removal

Appendix A solves the intertemporal optimization problem. It shows that the optimal rate of consumption is constant over time, $x_t^* = 1 - \beta\,\kappa$, and that the shadow value of the fossil resource stock, denoted by $\varphi_{R,t}$, monotonically grows over time according to Hotelling's (1931) rule, $\varphi_{R,t} = \beta^{-t}\varphi_{R,0}$. In the following, we summarize the results on optimal CDR deployment.

Proposition 1. *The optimal level of CDR deployment for reservoir i is given by*

$$G_{i,t}^* = f_i'^{-1}\left(\frac{\beta\,\xi_0\,[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}]_{1,1} - \beta\,\xi_0\,[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}]_{1,i}}{\beta\,\xi_0\,\,[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}]_{1,1} + (1-\beta\,\kappa)\beta^{-t}\varphi_{R,0}}\right), \tag{9}$$

where $[\cdot]_{1,1}$ denotes the first, and $[\cdot]_{1,i}$ denotes the i^{th} element of the first column of the inverted matrix in square brackets. Note that the inverse of the marginal cost function is expressed by $f_i'^{-1}$ and that $[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}]_{1,1} > [(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}]_{1,i}$.

Proof. See Appendix B. □

Optimal CDR deployment is a function of constant model parameters, and the endogenously determined shadow value of the resource stock, which monotonically grows over time. Since $f_i'(G_{i,t})$ is an increasing function, also its inverse $f_i'^{-1}$ is an increasing function. Thus, optimal CDR immediately starts with its maximum level and then monotonically declines over time.

The interpretation of the carbon dynamics contributions follows Traeger (2018): The term $[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}]_{1,1}$ characterizes the discounted sum of carbon persisting in and returning to the atmospheric carbon stock in all future periods. The term

$[(\mathbf{1} - \beta\,\mathbf{\Phi})^{-1}]_{1,i}$ characterizes the long-term contribution to the atmospheric carbon reservoir from carbon that is currently stored in reservoir i.

The numerator in equation (9) shows the marginal benefit of the new technology. CDR reduces the marginal damage of emissions as it allows to remove carbon from the atmosphere and store it in a less damaging reservoir i. The denominator shows the marginal cost of fossil energy. It captures the opportunity cost of the resource and the marginal damage that it creates.

The magnitude of the benefit from CDR is determined by the difference in the carbon dynamics contributions of the atmosphere and reservoir i. A decrease in the carbon persistence of reservoir i increases its carbon dynamics contribution as more carbon eventually finds its way into the atmosphere. This decreases the marginal benefit of CDR, and hence G_t^* declines. In contrast, an increase in the climate change damage parameter ξ_0 or an increase in the atmospheric carbon dynamics contribution $[(\mathbf{1} - \beta\,\mathbf{\Phi})^{-1}]_{1,1}$ raises the marginal damage of emissions and makes CDR technologies more attractive.

3.2 Emissions and energy input

Using the solution for CDR deployment allows to derive the optimal levels for emissions, and net energy input.

Proposition 2. *Optimal carbon emissions into the atmosphere are given by*

$$E_t^* = \frac{v}{\beta\,\xi_0\,[(\mathbf{1} - \beta\,\mathbf{\Phi})^{-1}]_{1,1} + (1-\beta\,\kappa)\beta^{-t}\varphi_{R,0}} + \sum_{i=2}^{r=4} f_i\left(G_{i,t}^*\right), \tag{10}$$

with optimal CDR deployment $G_{i,t}^*$ as defined in equation (9).

Proof. See Appendix C. □

Optimal emissions are given by the sum of two terms. The first term captures the marginal benefit (numerator) and the marginal cost (denominator) from fossil energy. The term monotonically declines over time as the shadow value of the fossil resource increases. The second term shows the total cost of CDR deployment (measured in energy units). According to Proposition 1 optimal deployment monotonically declines over time, and thus optimal emissions decline over time as well.

An increase in $\varphi_{R,0}$ makes the fossil resource a more expensive input for production, and decreases both terms in equation (10). An increase in the carbon dynamics contribution $\left[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}\right]_{1,i}$ increases the marginal damage from reservoir i. As a result CDR deployment declines, and thus optimal emissions are lower. The outcome of an increase in $\left[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}\right]_{1,1}$ and ξ_0 is ambiguous as there are two opposing effects. It decreases the first term in equation in (10) but leads to a higher level of CDR which increases the second term.

Using the solutions $G^*_{i,t}$ and E^*_t allows to solve for optimal net energy input I^*_t,

$$I^*_t = \frac{v}{\beta\,\xi_0\,\left[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}\right]_{1,1} + (1-\beta\,\kappa)\beta^{-t}\varphi_{R,0}}. \tag{11}$$

Net energy is defined as the difference between fossil energy and the energy spent on CDR. It is therefore equivalent to the first term in equation (10). Net energy input increases in the energy share v, and decreases in climate change damages ξ_0, the initial resource shadow value $\varphi_{R,0}$, and the carbon dynamics contribution $\left[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}\right]_{1,1}$.

In order to gain insides on what changes due to the introduction of CDR, We specify an alternative scenario by removing the option of CDR from the climate-economy model in section 2. We denote the variables of the alternative model without CDR by a tilde. In the following, We show that the introduction of CDR influences the initial shadow value of the nonrenewable resource, and analyze how this affects net energy input, and net emissions. We discuss the implications of CDR for E^*_t in the subsequent section.

Proposition 3. *CDR increases the shadow value of the fossil resource, and decreases net energy input and net emissions.*

Proof. See Appendix D. □

CDR increases the value of the fossil fuel resource as it creates an additional option to mitigate the negative effects from carbon emissions, and thus reduces the social costs of using fossil fuels. Due to the linear-in-states property of the model, there is no direct effect of CDR on the marginal damage of carbon emissions. As a result, the net effect of CDR on the cost of the fossil resource is positive, and thus net energy input declines, $\Delta I_t^* \equiv I_t^* - \tilde{I}_t^* < 0$.

Next, We compare how net emissions differ between both model types. The difference is given by

$$\begin{aligned}\Delta E_t^{net^*} &\equiv E_t^{net^*} - \tilde{E}_t^{net^*} \\ &= I_t^* - \tilde{I}_t^* + \sum_{i=2}^{r} \left(f_i(G_{i,t}^*) - G_{i,t}^* \right) < 0,\end{aligned}$$

since $\tilde{I}_t^* > I_t^*$ and $f_i(G_{i,t}^*) \leq G_{i,t}^*$.

CDR leads to lower net emission over the entire time path. This result is driven by two effects. First, as already shown CDR lowers net energy input, and second, the cost of CDR is lower than the cost of mitigation (both measured in energy units).

3.3 Social cost of carbon

This section derives the SCC for all reservoir types and explores how CDR influences the optimal carbon tax. Due to the linear-in-states property of the model the marginal damage for each reservoir type is independent of its stock size. This leads to the following result.

Proposition 4. *CDR leaves the structure of the atmospheric SCC (optimal carbon tax) unchanged. The SCC for reservoir i is proportional to net output,*

$$SCC_{Mi} = Y_t^{net}\,\xi_0\,\left[(\mathbf{1} - \beta\,\mathbf{\Phi})^{-1}\right]_{1,i}. \tag{12}$$

As defined above, $[\cdot]_{1,i}$ denotes the i^{th} element of the first column of the inverted matrix in square brackets.

Proof. See Appendix E. □

The persistence of carbon differs between reservoir types such that each reservoir has its own SCC. For example, for the DICE carbon cycle the carbon dynamics contribution of the deep ocean is smaller than the carbon dynamics contribution of the shallow ocean. This leads to the following ordering: $SCC_{M1} > SCC_{M2} > SCC_{M3}$. The carbon dynamics contribution of the geological reservoirs depend on the rates of decay to the atmospheric carbon stock. If it is a secure deposit and the decay rate is zero, then its reservoir specific SCC is zero.

Deriving the atmospheric SCC (optimal carbon tax) for the alternative model specification without CDR leads to the same result as in equation (12). The availability of CDR leaves the analytic structure of the atmospheric SCC unchanged. This result is driven by two crucial assumptions of analytic IAMs. First, utility is a logarithmic function of consumption, and second, climate change damages have an exponential impact on output. These two assumptions ensure that the climate-economy model is linear-in-states and can be solved by a linear affine value function (Karp, 2017). The linear-in-states property implies that the marginal damage from an additional unit of carbon in the atmosphere is constant and does not depend on the atmospheric carbon concentration. Hence, removing a unit of carbon from the atmosphere has no effect on the marginal damage, and the atmospheric SCC. This is different for other geoengineering measures such as stratospheric aerosol injections (Meier and Traeger, 2021).

Next, We analyze how the level of the SCC is affected by the availability of CDR, compared to the situation without CDR. According to Proposition 3 CDR leads to a lower net energy input over the entire time path. As a result initial output declines. Since the initial atmospheric carbon concentration and initial climate change damages are equivalent for both model types, initial net output decreases as well. This lowers the initial level of the atmospheric SCC, and therefore rises the level of emissions in the beginning. However, as CDR is an option it must increase net output eventually as otherwise it would no be used. Thus, there must exist a period in the future in which the SCC is higher compared to the model without CDR. The interpretation of this result is straight forward. As climate change damages are measured in percent of output, an increase in Y_t also increases the money-measured welfare loss from global warming. In other words: The better off the economy is, the more the economy loses from climate change. The next section quantifies this effect.

4 Quantitative analysis

This section illustrates the previous theoretical findings. It provides a calibration of the climate-economy model for a high and low-cost scenario of oceanic CDR, and compares the results to the alternative model specification without CDR.

4.1 Climate-economy model without CDR

The simulation starts in t = 2010 and ends in t = 2200 with one period representing ten years, which is a standard in the literature. Economic growth is driven by increasing total factor productivity $A_{0,t}$, which develops exogenously over time according to

$$A_{0,t} = A_0 \left(1 + w\right)^t. \tag{13}$$

The growth rate of total factor productivity is assumed to be 2 percent per year, $w = 0.02$. The initial population is set to 6.9 billion and assumed to grow logistically over time to a maximum of 11 billion in 2200 as in Gerlagh and Lsiki (2018). Output for the initial decade is set to 700 trillion (tn) USD. I use the same shares of capital, $\alpha = 0.3$, and net energy, $v = 0.04$, as in Golosov et al. (2014). The utility discount rate is set to 1.4 percent per year (Traeger, 2018). The given parameter set implies an optimal constant savings rate of $s \approx 0.25$. The initial capital stock is assumed to be 135 trillion USD, approximately the output of two years, and fully depreciates over the course of a decade. We use the carbon cycle from DICE 2013 (Nordhaus and Sztorc, 2013), and the climate change damage parameter $\xi_0 = 5.3 \text{x} 10^{-5}$ from Golosov et al. (2014). The pre-industrial carbon stock is set to 600 GtC. The carbon concentration for the first decade is set to 830.4 GtC yielding initial climate change damages of $D_0 = 1.2$ percent.

Assuming emissions of 86.7 GtC for the first decade (Gerlagh and Lsiki, 2018) allows to solve for the initial level of total factor productivity, and delivers $A_0 = 38$. We then calibrate the initial resource stock such that it matches the initial level of emissions. This implies an initial fossil fuel stock size of 793.25 GtC. Table 1 summarizes the model parameters and initial stock values.

Table 1: Parameter values

K_0	N_0	R_0	κ	v	β	w	A_0	ξ_0
135	6.9	793.25	0.3	0.04	0.986	0.02	38.02	$5.3 \text{x} 10^{-5}$
trillion USD	billion	GtC			1/year	1/year		1/GtC

Figure 1 shows the outcome of the standard model without CDR. The fossil resource is scarce and used up over the time horizon. Emissions start at 86 GtC per decade and monotonically decline over time as the shadow price of the resource increases. Damages start at 1.2 percent of global output and increase up to around 3 percent by the year 2100. Afterwards, damages start to decline as less energy is used and more carbon is taken up by the oceans. Relative net production (GDP) rises over time due to the growth of total factor productivity. The atmospheric SSC starts at around 45 USD/tCO_2 and increases up to around 800 USD/tCO_2 by the year 2100. All these results are very much in line with results of common IAMs (e.g. Golosov et al., 2014).

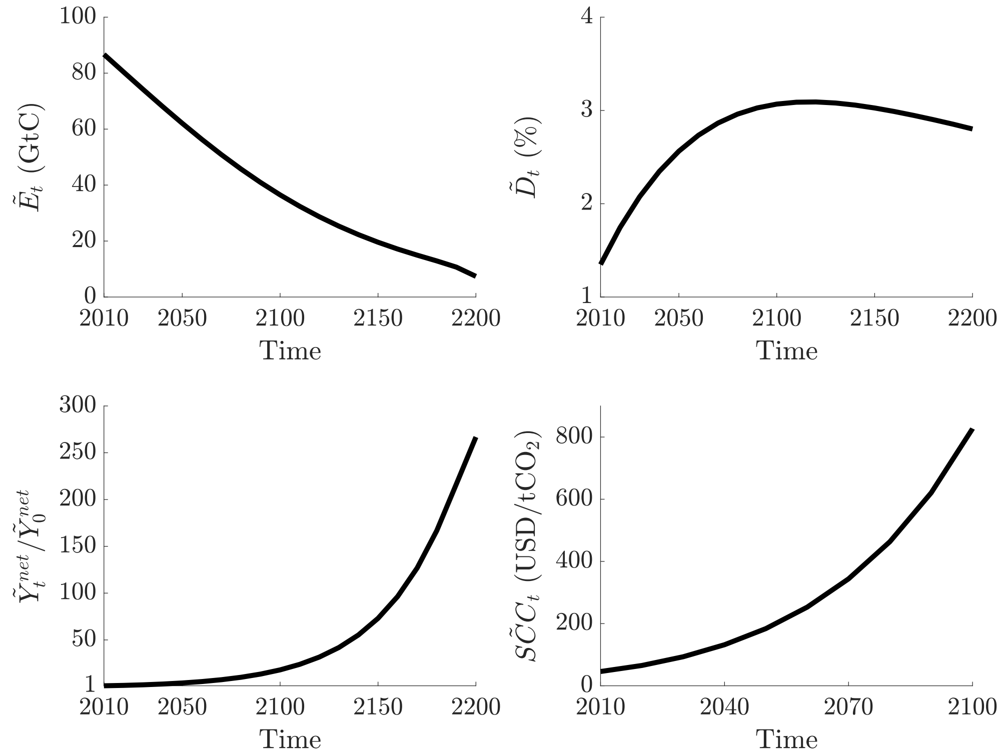

Figure 1: The graph shows emissions per decade ($\tilde{E}_t$), damages ($\tilde{D}_t$), relative net output ($\tilde{Y}_t^{net}/\tilde{Y}_0^{net}$), and the social cost of carbon ($S\tilde{C}C_t$) for the calibrated standard climate-economy model without CDR.

4.2 Climate-economy model with oceanic CDR

This section introduces the option of oceanic CDR and explores how it affects the outcome of the standard climate-economy model. Cost estimates for the storage of carbon in the oceans are still uncertain and vary widely. IPCC (2005) estimates the cost for oceanic storage between 22 and 114 USD/tC. Rickels et al. (2018) consider a convex cost function with a broad parameter range for the quadratic cost term to account for uncertainty about the cost of large-scale deployment.

To capture the cost uncertainty for oceanic CDR we consider a low and high-cost scenario. For the low-cost case, the cost function for CDR is given by

$$f_l(G_t) = g_l\, G_t^2, \tag{14}$$

with parameter g_l to be calibrated. As a point of reference, we use the linear quadratic cost function from Rickels et al. (2018) and combine it with the lower bound cost estimate for oceanic storage of 22 USD/tC from IPCC (2005), which leads to

$$F(G_t) = 0.022\, G_t + 0.01833\, G_t^2. \tag{15}$$

CDR deployment G_t is measured in GtC and $F(G_t)$ shows the costs in trillion USD (tn USD). We calibrate the cost function $f_l(G_t)$ to equation (15) for the initial time period. Minimizing the squared difference over the interval $G_t \in (0, 18.5)$ yields $g_l = 0.056$. We choose this interval since for $G_t \geq 18.5$ the cost of CDR is higher than the cost of mitigation. Figure 2 shows the quality of the fit, and the cost of mitigation in trillion USD. For the high-cost scenario, we consider the upper bound of previous estimates. As the upper bound cost estimate is expected to surpass the lower bound cost estimate by a factor of five (IPCC, 2005), we assume $g_h = 5 \times g_l$. Due to the assumption of a quadratic cost function the level of CDR will still be positive but considerably lower than in the low-cost scenario. Figures 3 and 4 show how the results change due to the introduction of CDR. The black solid lines show the results for the low-cost scenario and the dotted green lines show the outcome for the high-cost scenario.

The simulation illustrates the analytic results from the previous section. In the first decade, in the low-cost case around 4.5 GtC are removed from the atmosphere and stored in the deep ocean. In the high-cost scenario, CDR deployment is considerably lower with only 1 GtC in the first decade. As described in Propositions 1 and 2 CDR deployment and emissions monotonically decline over time. In line

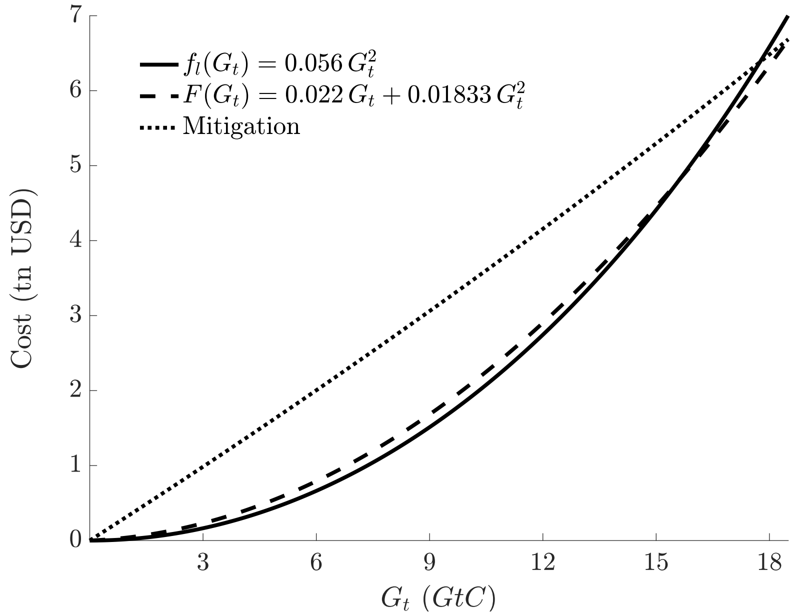

Figure 2: The graph shows the calibrated cost function $f_l(G_t)$ (solid line), the cost estimate based on Rickels et al. (2018) and IPCC (2005) (dashed line), and the cost of mitigation (dotted line).

with Proposition 3, net emissions and net energy input is lower over the entire time horizon compared to the model without CDR. In both scenarios emissions are first higher and then lower than in the model without CDR. In the low cost scenario the difference is more pronounced.

In the low-cost case, CDR reduces the atmospheric carbon concentration by 20 GtC in 2125 and damages are lower by around 0.1 percentage points of output. Towards the end, the negative effect on the atmospheric carbon concentration and climate damages wears off as CDR deployment goes to zero and more and more carbon has cycled back from the oceans. In the high-cost case, the negative effect on atmospheric carbon is minor and only decreases damages by around 0.01 percent.

The numerical simulation also allows to assess how strongly net output and the atmospheric SCC (optimal carbon tax) are affected by the introduction of CDR. As already discussed in the theoretical part of the paper initial net output declines as CDR becomes available. Figure 4 shows that this effect is rather small. Net output declines by 0.025 percent in the low-cost scenario. Afterwards, the effect on net

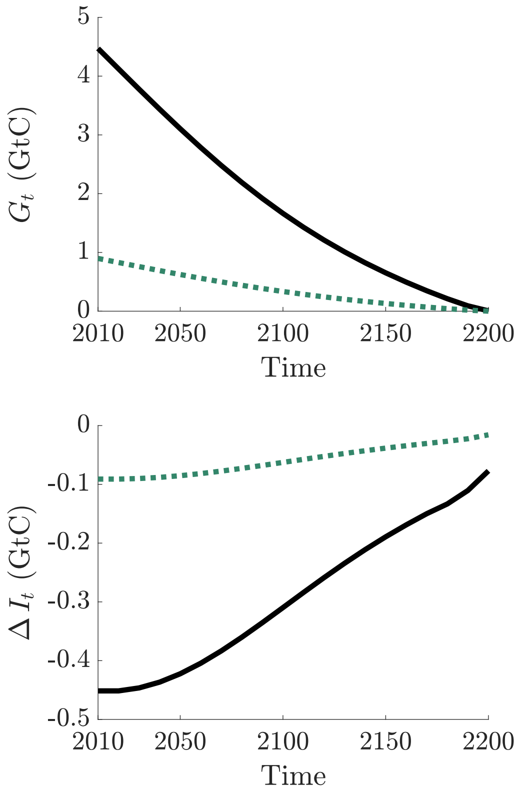

Figure 3: The graph shows the optimal deployment of oceanic CDR (G_t) per decade and the difference in emissions ($\Delta\, E_t$), net energy input ($\Delta\, I_t$), and net emissions ($\Delta\, E_t^{net}$) compared to the outcome of the standard model without CDR for the low cost (black solid lines), and high cost scenario (green dotted lines).

output becomes positive and grows until 2125 to around 0.11 percent. Similar to net output, the SCC is first lower and then higher. The economy first emits more and then less. The simulation shows that the effect of CDR on the SCC is minor. By 2100 the SCC is only higher by 3 USD/tCO_2 compared to the model without CDR.

5 Summary and conclusions

The paper introduces the option of carbon dioxide removal (CDR) and storage in different reservoir types into an analytic climate-economy model and compares the results to a model variant without CDR. The analytic model shows that the

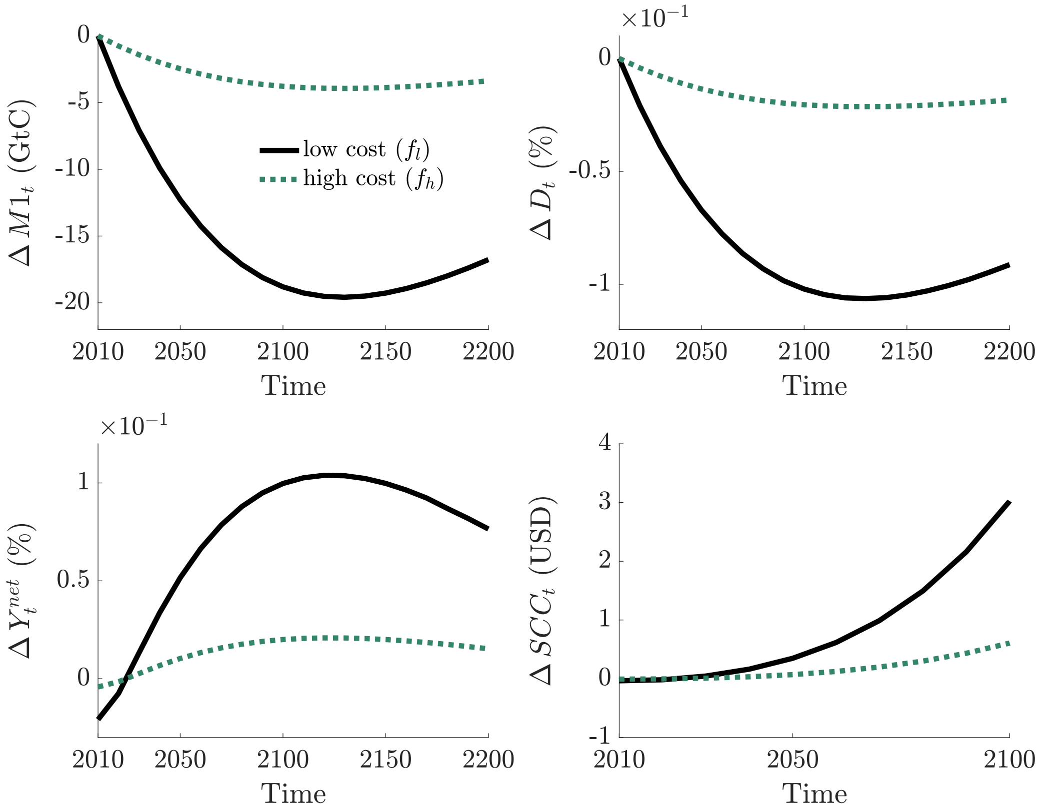

Figure 4: The graph shows the difference in atmospheric carbon concentration ($\Delta M_{1,t}$), climate change damages (ΔD_t), net output (ΔY_t^{net}), and the social cost of carbon (ΔSCC_t) compared to the outcome of the standard model without CDR for the low cost (black solid lines), and high cost scenario (green dotted lines).

availability of CDR alters the level of the SCC. However, the quantitative analysis suggests that this effect is negligible. In the low-cost scenario, CDR increases initial emissions by around 0.6 GtC, which is equivalent to around 0.7 percent of total carbon emissions. Thus, with an optimal policy in place the introduction of CDR has hardly any effect on mitigation incentives. The model suggests that CDR is needed on top of traditional mitigation efforts.

Furthermore, the paper provides basic implications for the optimal implementation of CDR technologies. One option that has been proposed in the literature is the introduction of a differentiated carbon tax (Rickels and Lontzek, 2012). This paper presents a simple formula for the reservoir specific carbon tax, and characterizes its components. Another suggestion for the optimal implementation of CDR is the introduction of carbon credits (Chomitz and Lecocq, 2004; Sedjo and Marland, 2003), for which this paper also offers a simple way to calculate it. The analytical structure with the different social cost for the various boxes allows assessing a broad variety of marine CDR options by considering different boxes in the carbon cycle. This provides the basis for the analysis of decentralized and potentially non-cooperative CDR policies. In a further extension we will renewable energies which would reduce the cost of CDR.

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Appendices

A Solving the linear-in-states model

For the proof of the linear-in-states property I follow Traeger (2018). The consumption rate can be written as

$$x_t = \frac{C_t}{Y_t \, [1 - D_t \, (M_{1,t})]},$$

such that

$$\log C_t = \log x_t + \log A_{0,t} + \kappa \log K_t + (1 - \kappa - v) \log N_{0,t} + v \log I_t - \xi_0 \, (M_{1,t} - M_1^{pre}).$$

I transform the optimization problem into its dynamic programming form (Bellman equation)

$$\begin{aligned} V(k_t, \boldsymbol{M}_t, R_t, t) = \max_{x_t, E_t, G_t} \Bigg\{ & \log x_t + \log A_{0,t} + \kappa \log K_t + (1 - \kappa - v) \log N_{0,t} \\ & + v \log I_t(E_t, G_{i,t}) - \xi_0 \, (M_{1,t} - M_1^{pre}) + \beta \, V(k_{t+1}, \boldsymbol{M}_{t+1}, R_{t+1}, t+1) \Bigg\}, \end{aligned}$$

where $k_t = \log K_t$ with the equation of motion

$$k_{t+1} = \log A_{0,t} + \kappa \log K_t + (1 - \kappa - v) \log N_{0,t} + v \log I_t - \xi_0 \, (M_{1,t} - M_1^{pre}) + \log(1 - x_t). \tag{16}$$

To solve the intertemporal optimization problem, I use the following guess for the value function

$$V(k_t, \boldsymbol{M}_t, R_t, t) = \varphi_k \, k_t + \boldsymbol{\varphi}_M^T \, \boldsymbol{M}_t + \varphi_{R,t} \, R_t + \varphi_t, \tag{17}$$

where φ is used to denote the shadow values for the different states, and ^{T} denotes the transpose of a vector of shadow values.

Inserting the trial solution and the next periods states (equations 2, 3, and 16) into the Bellman equation delivers

$$\begin{aligned}
&\varphi_k\, k_t + \boldsymbol{\varphi}_M^T\, \boldsymbol{M}_t + \varphi_{R,t}\, R_t + \varphi_t \\
&= \max_{x_t, E_t, G_{i,t}} \Bigg\{ \log x_t + \log A_{0,t} + \kappa\, k_t + (1-\kappa-v) \log N_{0,t} + v \log I_t(E_t, G_{i,t}) - \xi_0\, (M_{1,t} - M_1^{pre}) \\
&+ \beta\, \varphi_k\, \big(\log A_{0,t} + \kappa\, k_t + (1-\kappa-v) \log N_{0,t} + v \log I_t(E_t, G_{i,t}) - \xi_0\, (M_{1,t} - M_1^{pre}) + \log(1-x_t) \big) \\
&\qquad + \beta\, \boldsymbol{\varphi}_M^T\, (\boldsymbol{\Phi M}_t + \boldsymbol{E}_t) + \beta\, \varphi_{R,t+1}\, (R_t - E_t) + \beta\, \varphi_{t+1} \Bigg\}. \qquad (18)
\end{aligned}$$

First order conditions. Maximizing the right hand side over x_t yields

$$\frac{1}{x_t} - \beta\, \varphi_k \frac{1}{1-x_t} = 0 \quad \Longrightarrow \quad x_t^* = \frac{1}{1+\beta\, \varphi_k}. \qquad (19)$$

Next, I find the first order condition for CDR deployment for reservoir i

$$-v(1+\beta\, \varphi_k)\, \frac{f_i'(G_{i,t})}{I_t} = \beta(\varphi_{M1} - \varphi_{Mi}), \qquad (20)$$

and the first order condition for emissions

$$v(1+\beta\, \varphi_k)\, \frac{1}{I_t} = \beta(\varphi_{R,t+1} - \varphi_{M1}). \qquad (21)$$

Inserting (21) into (20) and solving for $G_{i,t}$ leads to

$$G_{i,t}^* = f_i'^{-1} \left(\frac{\varphi_{M1} - \varphi_{Mi}}{\varphi_{M1} - \varphi_{R,t+1}} \right), \qquad (22)$$

where the inverse of the marginal cost function is denoted by $f_i'^{-1}$. Summing up

CDR deployment over all reservoir types yields

$$G^*_{1,t} = \sum_{i=2}^{r} f_i'^{-1}\left(\frac{\varphi_{M1} - \varphi_{Mi}}{\varphi_{M1} - \varphi_{R,t+1}}\right)$$

Using (22) and solving for optimal emissions yields

$$E^*_t = \frac{v(1+\beta\,\varphi_k)}{\beta(\varphi_{R,t+1} - \varphi_{M1})} + \sum_{i=2}^{r} f_i\left(f_i'^{-1}\left(\frac{\varphi_{M1} - \varphi_{Mi}}{\varphi_{M1} - \varphi_{R,t+1}}\right)\right). \tag{23}$$

First order conditions deliver optimal controls x^*_t, E^*_t, and $G^*_{i,t}$ which are independent of the states.

Using E^*_t and $G^*_{i,t}$ one can solve for the optimal net energy input I^*_t.

$$I^*_t = E^*_t - \sum_{i=2}^{r} f_i(G^*_{i,t}) = \frac{v(1+\beta\,\varphi_k)}{\beta(\varphi_{R,t+1} - \varphi_{M1})}. \tag{24}$$

Inserting the optimal controls into (18) and arranging terms with respect to their states yields

$$\begin{aligned}
\varphi_k\,k_t + \boldsymbol{\varphi}_M^T\,\boldsymbol{M}_t + \varphi_{R,t}\,R_t + \varphi_t = \Big[(1+\beta\,\varphi_k)\kappa\Big]k_t + \Big[\beta\,\boldsymbol{\Phi}\boldsymbol{\varphi}_M^T - (1+\beta\,\varphi_k)\xi_0\,\boldsymbol{e}_1^T\Big]\boldsymbol{M}_t \\
+\Big[\beta\,\varphi_{R,t+1}\Big]R_t + \log\,x^*_t + \beta\,\varphi_k\,\log(1-x^*_t) + (1+\beta\,\varphi_k)\log A_{0,t} + (1+\beta\,\varphi_k)(1-\kappa-v)\log N_{0,t} \\
+(1+\beta\,\varphi_k)v\log I^*_t + (1+\beta\,\varphi_k)\xi_0\,M_1^{pre} + \beta\,\varphi_{M1}(E^*_t + E^{\text{exo}}_t - G^*_{1,t}) + \beta\,\varphi_{M2}\,G^*_{2,t} + ... + \\
+\beta\,\varphi_{Mr}\,G^*_{r,t} - \beta\,\varphi_{R,t+1}\,E^*_t + \beta\,\varphi_{t+1}.
\end{aligned} \tag{25}$$

Given the optimal controls the maximized Bellman equation is linear in all states.

Shadow values. Coefficient matching with respect to capital, k_t, yields

$$\varphi_k = (1+\beta\,\varphi_k)\kappa \quad \Leftrightarrow \quad \varphi_k = \frac{\kappa}{1-\beta\,\kappa} \tag{26}$$

Inserting φ_k into equation (19) yield the optimal consumption rate $x^*_t = 1 - \beta\,\kappa$.

I match the coefficients of each state from both sides of the equation, which leads to

$$\boldsymbol{\varphi}_M^T = -\xi_0 \,(1 + \beta\,\varphi_k)\, \boldsymbol{e}_1^T \,[\mathbf{1} - \beta\,\boldsymbol{\Phi}]^{-1}$$

Using (26) the vector of shadow prices turns to

$$\boldsymbol{\varphi}_M^T = -\xi_0 \,\frac{1}{1-\beta\,\kappa}\, \boldsymbol{e}_1^T \,[\mathbf{1} - \beta\,\boldsymbol{\Phi}]^{-1} \tag{27}$$

Coefficient matching with respect to the resource stock yields

$$\varphi_{R,t} = \beta\,\varphi_{R,t+1} \quad \Leftrightarrow \quad \varphi_{R,t} = \beta^{-t}\varphi_{R,0} \quad \text{(Hotelling's rule)}. \tag{28}$$

The initial resource values $\varphi_{R,0}$ depend on the set up of the economy, including assumptions about production and the energy sector. Given the coefficients and the optimal rate of consumption equation (25) turns to the following condition:

$$\begin{aligned}\varphi_t - \beta\,\varphi_{t+1} = \log\, x_t^* + \beta\,\varphi_k\, \log(1-x_t^*) + (1+\beta\,\varphi_k)\log A_{0,t} + (1+\beta\,\varphi_k)(1-\kappa-v)\log N_{0,t} \\ + (1+\beta\,\varphi_k) v \log I_t^* + (1+\beta\,\varphi_k)\xi_0\, M_1^{pre} + \beta\,\boldsymbol{\varphi}_M^T\,\boldsymbol{E}_t^* - \beta\,\varphi_{R,t+1}\,E_t^*\end{aligned}$$

This condition will be satisfied by picking the sequence $\varphi_0, \varphi_1, \varphi_2, ...$. The additional condition $\lim_{t\to\infty} \beta^t V(\cdot) = 0 \Rightarrow \lim_{t\to\infty} \beta^t \varphi_t = 0$ pins down this initial value φ_0.

B Proof of Proposition 1

Inserting the solutions for the shadow values, equations (26) to (28), into (22) yields

$$G_{i,t}^* = f_i'^{-1}\left(\frac{\beta\,\xi_0\,[(\mathbf{1}-\beta\,\boldsymbol{\Phi})^{-1}]_{1,1} - \beta\,\xi_0\,[(\mathbf{1}-\beta\,\boldsymbol{\Phi})^{-1}]_{1,i}}{\beta\,\xi_0\,\,[(\mathbf{1}-\beta\,\boldsymbol{\Phi})^{-1}]_{1,1} + (1-\beta\,\kappa)\beta^{-t}\varphi_{R,0}}\right), \tag{29}$$

where $[\cdot]_{1,1}$ denotes the first, and $[\cdot]_{1,i}$ denotes the i^{th} element of the first column of the inverted matrix in square brackets. Note that $\left[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}\right]_{1,1} > \left[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}\right]_{1,i}$.

C Proof of Proposition 2

Inserting the solutions for the shadow values, equations (26) to (28), into (23) yields

$$E_t^* = \frac{v}{\beta\,\xi_0\,\left[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}\right]_{1,1} + (1-\beta\,\kappa)\beta^{-t}\varphi_{R,0}} + \sum_{i=2}^{r} f_i\left(G_{i,t}^*\right), \tag{30}$$

where

$$G_{i,t}^* = f_i'^{-1}\left(\frac{\beta\,\xi_0\left[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}\right]_{1,1} - \beta\,\xi_0\left[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}\right]_{1,i}}{\beta\,\xi_0\,\left[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}\right]_{1,1} + (1-\beta\,\kappa)\beta^{-t}\varphi_{R,0}}\right).$$

D Proof of Proposition 3

Consider the climate-economy model from section 2 without the option of CDR, and let the variables of this model specification be denoted by a tilde.

From the first order condition (20) it follows that optimal emissions without the option of CDR are given by

$$\tilde{E}_t^* = \frac{v}{\beta\,\xi_0\,\left[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}\right]_{1,1} + (1-\beta\,\kappa)\beta^{-t}\tilde{\varphi}_{R,0}}. \tag{31}$$

The only endogenous term in equation (31) is the initial shadow value of the resource stock, which is denoted by $\tilde{\varphi}_{R,0}$. In both model specifications, the size of the resource stock is the same and will be used up eventually. Therefore,

$$R_0 = \sum_{t=0}^{\infty} E_t^* = \sum_{t=0}^{\infty} \tilde{E}_t^*.$$

Using equations (30) and (31), and rearranging leads to

$$\sum_{t=0}^{\infty}\sum_{i=2}^{r} f_i(G^*_{i,t}) = \sum_{t=0}^{\infty}\left(\frac{v}{\beta\,\xi_0\ [(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}]_{1,1} + (1-\beta\,\kappa)\beta^{-t}\tilde{\varphi}_{R,0}} - \frac{v}{\beta\,\xi_0\ [(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}]_{1,1} + (1-\beta\,\kappa)\beta^{-t}\varphi_{R,0}}\right).$$

If there exists at least one point in time where $\sum_{i=2}^{r} f_i(G^*_{i,t}) > 0$, the left term of the equation is positive, and thus $\tilde{\varphi}_{R,0} < \varphi_{R,0}$. From this it directly follows that $\Delta\, I^*_t \equiv I^*_t - \tilde{I}^*_t < 0$.

Comparing net emissions with and without the option of CDR yields

$$\begin{aligned}\Delta E^{net^*}_t &\equiv E^{net^*}_t - \tilde{E}^{net^*}_t \\ &= I^*_t - \tilde{I}^*_t + \sum_{i=2}^{r}\left(f_i(G^*_{i,t}) - G^*_{i,t}\right) < 0,\end{aligned}$$

since $\tilde{I}^*_t > I^*_t$ and $f_i(G_{i,t}) \leq G^*_{i,t}$.

E Proof of Proposition 4

The SCC is the negative of the shadow value of carbon reservoir i expressed in money-measured consumption units,

$$\begin{aligned}SCC_{Mi} &= -(1-\beta\,\kappa)Y^{net}_t\,\varphi_{Mi} \\ &= Y^{net}_t\,\xi_0\ \left[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}\right]_{1,i},\end{aligned}$$

where again $[\cdot]_{1,i}$ denotes the i^{th} element of the first column of the inverted matrix in square brackets.